# Design and Performance Evaluation of an Asymptotically Optimal Backoff Algorithm for IEEE 802.11 Wireless LANs

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### Abstract

This paper presents and evaluates a distributed mechanism for the contention control in IEEE 802.11 Wireless LANs. Specifically, our mechanism named Asymptotically Optimal Backoff (AOB), dynamically adapts the backoff window size to the current load. AOB guarantees that an IEEE 802.11 WLAN asymptotically (i.e. for a large number of active stations) achieves its optimal channel utilization. The proposed mechanism merges the ideas on adaptive backoff presented in [2] with some properties derived from the IEEE 802.11 capacity analysis (see [3]). AOB can be used on top of the standard 802.11 access mechanism without requiring any modification to the standard or additional hardware. The AOB mechanism adapts the backoff to the network contention level by using two simple load estimates: the slot utilization and the average size of transmitted frames. These estimates are simple and can be obtained with no additional costs or overheads. The performance of the IEEE 802.11 protocol with or without the AOB mechanism is investigated in the paper via simulation. Simulative results indicate that our mechanism is very effective and brings the utilization of the system close to the optimal level for a wide range of load and network configurations.

# **1. Introduction**

In WLANs, the medium access control (MAC) protocol is the main element that determines the efficiency in sharing the limited communication bandwidth of the wireless channel. In this paper we focus on the efficiency of the IEEE 802.11 standard for wireless LANs.

The IEEE 802.11 access scheme incorporates two access methods: Distributed Coordination Function (DCF) for asynchronous, contention-based, distributed access to the channel, and Point Coordination Function (PCF) for centralized, contention-free accesses [8, 17]. We will concentrate our study on DCF. The DCF method adopts an access scheme belonging to the class of the CSMA/CA MAC protocols [4,

8, 12, 19]. The distributed congestion reaction adopted in the 802.11 DCF is obtained with a variable time-spreading of the users' accesses. A channel utilization wastage is caused both by collisions and by the idle periods introduced by the spreading of accesses. To optimize the channel utilization the access protocol should balance these two conflicting costs [3,9]. Since these costs change dynamically, depending on the network load, the access protocol should be adaptive to congestion variations in the system. Such an adaptive behavior is currently obtained in the IEEE 802.11 protocol by adopting a binary exponential backoff protocol [8, 11, 12]. Specifically, each user is not assumed to have any kind of knowledge about either the transmission result (success or collision), or the number of users in the system. Each station, to transmit a frame, accesses the channel within a random self-defined amount of time, whose mean size depends on the number of collisions previously experienced by the station for that frame. This policy has to pay the cost of collisions to increase the backoff time when the network is congested.

Several authors have investigated the enhancement of the IEEE 802.11 DCF MAC protocol to increase its performance when it is utilized in WLANs. In [5, 7], via a performance analysis, it is studied the tuning of the standard parameters. In [21], given the Binary Exponential Backoff scheme adopted by the Standard, solutions have been proposed for a better uniform distribution of accesses.

Trying to extend backoff protocols, a great amount of work has been done to study the information that can be obtained by observing the system's parameters [10, 14, 20]. Some studies try to approximate the knowledge about the number of users involved in the accesses by exploiting the history of the system. Example of such works (for the IEEE 802.11 DCF MAC protocol) relates to the attempt to make the reduction of contention adaptive and optimal by investigating the number of users in the system [1,3]. It is worth observing how this investigation could result expensive, difficult to obtain and subject to significant errors, especially in high contention situations.

In this paper we propose and evaluate a mechanism, Asymptotically Optimal Backoff (*AOB*), for improving the efficiency of the IEEE 802.11 standard protocol. This mechanism does not require any modification to the standard or additional hardware. The AOB mechanism adapts the backoff to the network contention level by using two simple and low-cost load estimates: the slot utilization and the average size of transmitted frames. AOB is based on the results derived by exploiting the analytical model of the IEEE 802.11 protocol presented in [1]. These results show that, given the average length of the transmitted frames, it exists a value for the channel utilization that maximizes the protocol capacity [6], this value is indicated as *optimal value* in the following. In addition, the optimal value is almost independent on the network configuration (number of active stations). AOB, by exploiting a

rough and low cost estimate of the average size of transmitted frames, guarantees that the channel utilization tends to the optimal value when the network is congested. To achieve this goal, AOB schedules the frames' transmission according to the IEEE 802.11 backoff algorithm but adds an additional level of control before a transmission is enabled. Specifically, when the channel utilization tends to exceed the optimal value, AOB forces a station to postpone the transmission already enabled by the standard backoff algorithm. The postponed transmission is rescheduled as in the case of a collision (i.e., the transmission is delayed of a further backoff interval). The proposed mechanism is applicable, in a transparent way, on top of the IEEE 802.11 DCF access mechanism. In this paper, via simulation, the AOB performance are deeply investigated and compared with the performance of the standard IEEE 802.11 DCF access scheme. This performance analysis indicates that: i) under light traffic conditions (i.e. few active stations), the AOB has almost no impact on the protocol performance; *ii*) under heavy traffic conditions, by adopting the AOB mechanism, the channel utilization is close to the (analytically defined) optimal level.

The work is organized as follows: in Section 2 we present a brief explanation of the IEEE 802.11 standard, and we sketch the critical aspects connected to the contention level of the system. In Section 3 the DCC mechanism proposed in [2] is summarized, while in Section 4 we sketch the capacity analysis presented in [3]. In Section 5 the AOB mechanism is defined. AOB implementation implementation issues and specifications are discussed in Section 6. In Section 7 AOB performance are investigated via simulation. Conclusions and future researches are outlined in Section 8.

# 2. IEEE 802.11 DCF Utilization

The basic access method in the IEEE 802.11 MAC protocol is the *Distributed Coordination Function* (DCF) which is a *Carrier Sense Multiple Access with Collision Avoidance* (CSMA/CA) MAC protocol. In addition to the DCF, the IEEE 802.11 also incorporates an alternative access method known as the *Point Coordination Function* (PCF) - an access method that is similar to a polling system and uses a point coordinator to determine which station has the right to transmit. In this section we only present the aspects of the DCF access method relevant for the scope of this paper. For the detailed explanation of the IEEE 802.11 standard we address interested readers to [8,15].

The DCF access method is based on a CSMA/CA MAC protocol that requires every station to perform a Carrier Sensing activity to determine the current state of the channel (idle or busy). If the medium is found to be busy, the station defers its transmission. Whenever the channel becomes idle for at least a Distributed Interframe Space time interval (DIFS), the station (re)starts its Basic Access mechanism. To avoid collisions, as soon as an idle DIFS is sensed on the channel, a Collision Avoidance mechanism is needed. The Collision Avoidance mechanism adopted in the IEEE 802.11 standard is based on a *Binary Exponential Backoff* scheme [8,11,12,13]. When the channel is idle, time is measured in constant length units (*Slot\_Time*) indicated as slots in the following.

The Binary Exponential Backoff scheme is implemented by each station by means of a parameter, named *Backoff Counter*, which maintains the number of empty slots the tagged station must observe on the channel before performing its own transmission attempt. At the time a tagged station needs to schedule a new transmission, it selects a particular slot among those of its *Contention Window*, whose size is maintained in the local parameter *CW\_Size*. Specifically, the backoff value is defined by the following expression [8]:

$$Backoff\_Counter = INT(Rnd() \cdot CW\_Size)$$
,

where Rnd() is a function which returns pseudo-random numbers uniformly distributed in [0..1].

The Backoff\_Counter is decreased as long as a slot time is sensed as idle, it is frozen when a transmission is detected, and reactivated after the channel is sensed as idle for at least a further DIFS. As soon as the Backoff Counter reaches the value Zero the station transmits its own frame. Positive acknowledgements are employed to ascertain a successful transmission. This is accomplished by the receiver (immediately following the reception of the data frame) which initiates the transmission of an acknowledgement frame (ACK) after a time interval *Short Inter Frame Space* (SIFS), which is less than DIFS.

If the transmission generates a collision<sup>1</sup>, the CW\_Size parameter is doubled for the new scheduling of the retransmission attempt thus obtaining a further reduction of contention. The Binary Exponential Backoff is then characterized by the expression that gives the dependency of the CW\_Size parameter by the number of *unsuccessful transmission attempts* ( $N_A$ ) already performed for a given frame. In [8] it is defined that the first transmission attempt for a given frame is to be performed with CW\_Size equal to the minimum value  $CW_Size_min$  (assuming low contention). After each unsuccessful (re)transmission of the same frame, the station doubles CW\_Size until it reaches the maximal value fixed by the standard, i.e.  $CW_Size_MAX$ , as follows:

 $CW\_Size(N\_A) = \min(CW\_Size\_MAX, CW\_Size\_\min \cdot 2^{(N\_A-1)})$ 

The increase of the CW\_Size parameter value after a collision is the reaction that the

<sup>&</sup>lt;sup>1</sup>A collision is assumed whenever the ACK from the receiver is missing

802.11 standard DCF provides to make the access mechanism adaptive to channel conditions.

# 2.1 IEEE 802.11 congestion reaction

Figure 1 shows simulation data regarding the channel utilization of a standard 802.11 system running in DCF mode, with respect to the contention level, i.e. the number of active stations with continuous transmission requirements.

The parameters adopted in the simulation, presented in Table 1, refer to the Frequency Hopping Spread Spectrum implementation [8].

Table 1: System's physical parameters								
parameter	value							
Number of Stations (M)	variable from 2 to 200							
CW_Size_min	16							
CW_Size_MAX	1024							
Channel transmission rate	2 Mb/s							
Payload size	Geometric distribution (parameter							
	q)							
Acknowledgement size	200 $\mu$ sec (50 Bytes)							
Header size	$136\mu\text{sec}$ (34 Bytes)							
SlotTime	$50\mu\mathrm{sec}$							
SIFS	28 µsec							
DIFS	128 µsec							
Propagation time	$< 1 \mu sec$							

Figure 1 plots the channel utilization versus the number of active stations obtained in asymptotic conditions, i.e. assuming that all the stations have always a frame to transmit. By analyzing the behavior of the 802.11 DCF mechanism some problems could be identified. Specifically, the results presented in the figure show that the channel utilization is negatively affected by the increase in the contention level.

These results can be explained as, in the IEEE 802.11 backoff algorithm, a station selects the initial size of the Contention Window by assuming a low level of congestion in the system. This choice avoids long access delays when the load is light. Unfortunately, this choice causes efficiency problems in bursty arrival scenarios, and in congested systems, because it concentrates the accesses in a reduced time window, and hence it may cause a high collision probability. In high-congestion conditions each station reacts to the contention on the basis of the collisions so far experienced while transmitting a frame. Every station performs its attempts blindly, with a late collision reaction performed (increasing CW\_Size). Each increase of the CW\_Size is obtained paying the cost of a collision. Furthermore, after a successful transmission the CW\_Size is set again to the

minimum value without maintaining any knowledge of the current contention level. To summarize the IEEE 802.11 backoff mechanism has two main drawbacks: i) the increase of the CW\_Size is obtained paying the cost of a collision, and ii) after a successful transmission no state information indicating the actual contention level is maintained.



Figure 1: Channel utilization of the IEEE 802.11 DCF access scheme

#### 3. Low-cost dynamic tuning of the backoff window size

The drawbacks of the IEEE 802.11 backoff algorithm, explained in the previous section, indicate the direction for improving the performance of a random access scheme, by exploiting early and meaningful information's concerning the actual state of congestion of the channel. The idea involves an estimate of the channel's congestion level, given by the utilization rate of the slots (*Slot Utilization*) observed on the channel by each station. The estimate of the Slot Utilization must be frequently updated. For this reason in [2] it was proposed an estimate that has to be updated by each station in every *Backoff interval*, i.e., the defer phase that precedes a transmission attempt. For the use we considered to make with, the Slot Utilization estimate has to satisfy only two conditions:

- values included in [0..1]: the Zero value should indicate that no slots observed in the backoff interval resulted as busy, while the value One should indicate that every slot available for transmission resulted as busy;
- <sup>°</sup> intermediate values should be distributed in the [0..1] interval, proportionally to the contention level (e.g. the rate of busy slots compared to the total number of



A simple and intuitive definition of the slot utilization estimate is then given by (see Figure 2):

$$Slot\_Utilization = \frac{Num\_Busy\_Slots}{Num\_Available\_Slots}$$

where *Num\_Busy\_Slots* is the number of transmission attempts<sup>2</sup> in the backoff interval, and *Num\_Available\_Slots* is the total number of slots available for transmission in the backoff interval.

In the 802.11 standard mechanism every station performs a Carrier Sensing activity and thus the proposed slot utilization  $(S_U)$  estimate is simple to obtain, with zero costs and overheads. The information required to estimate  $S_U$  are already available to an IEEE 802.11 station, with no additional hardware required.

It is interesting to observe how the slot utilization provides a lower bound for the actual contention level of the channel. In fact, as some stations may transmit in the same slot, it provides a lower bound to the effective number of stations trying to access the channel during the last observed backoff interval. If the value of the slot utilization is high (i.e. near to One), this implies that the last observed backoff interval was affected by a high level of contention on the channel.

# 3.1 The DCC mechanism

We have seen that the slot utilization estimate provides to each station an indication of the network contention level. This information can be utilized by each station to evaluate (before trying a "blind" transmission) the opportunity to perform or to defer its scheduled transmission attempt. In few words, the only reasonable behavior of a station that knows there are few possibilities for a successful transmission, is to defer its transmission attempt. Such a behavior can be achieved in an IEEE 802.11 network by exploiting the DCC mechanism proposed in [2]. According to DCC, each station controls its transmission attempts via a new parameter named *Probability of Transmission P\_T(...)* which constitutes the core of the proposed mechanism.

The P\_T parameter allows to realize a filtering of the accesses between the standard access scheme adopted by the system, and the physical layer, as shown in Figure 3. The value of this parameter is dependent on the contention level of the

<sup>&</sup>lt;sup>2</sup>It is worth noting that *Num\_Busy\_Slots* includes both successful transmissions and collisions.

channel. Each station, first estimates the slot utilization, then computes the Probability of Transmission value.  $P_T$  is used to evaluate the opportunity to perform a transmission on the shared channel. If the station decides to defer the transmission, it reschedules a new attempt, as in the case of a collision occurred (Figure 3). We now present the heuristic expression to evaluate the Probability of Transmission ( $P_T$ ) adopted in the proposed DCC mechanism [2]:

$$P_T(S_U) = 1 - S_U$$

where, by definition,  $S_U$  assumes values in the interval [0..1].



Figure 3: DCC and IEEE 802.11

The slot utilization is interpreted in DCC as an inhibition mechanism of the accesses, depending on the contention level it represents. However, we can observe how such a flat definition would conduct the system to fluctuate between two states, with slot utilization zero and one, i.e. no channel utilization and maximum contention, respectively. In fact, if the slot utilization is high, then every user would obtain a low  $P_T$  value, inducing a low slot utilization in the next future. This will cause every user to obtain a high  $P_T$  value, and a future high slot utilization. To avoid this harmful fluctuating behavior, the  $P_T$  definition has been extended by introducing a further local parameter. The idea is to partition the set of active stations in such a way that each stations' subset is associated with a different level of privilege to access the channel. This is achieved by including into the  $P_T$  definition the number of attempts already performed for the current frame ( $N_A$ ). The  $N_A$  parameter is used as an indicator of the dynamic level of privilege achieved by a station:

$$P_T(S_U, N_A) = 1 - S_U^{N_A}$$

The lowest privilege is given to stations performing, for a given frame, the first transmission attempt, while the privilege level linearly increases with the number of collisions experienced. To better understand what we obtain, we can observe the Figure 4.



Figure 4: Probability of Transmission

In this figure we show the Probability of Transmission curves for users with different numbers of attempts performed, and with respect to the  $S_U$  values estimated. Assuming a slot utilization near to zero, we can observe how each station, independently by its number of performed attempts, obtains a Probability of Transmission near to one. This means that the proposed mechanism has no effect on the system, and each user performs its accesses just like in the standard access scheme, without any additional contention control. This point is significant as it implies the absence of overhead introduced in low-load conditions. The differences in the user's behavior as a function of their levels of privilege (related to the value of the  $N_A$  parameter) appear when the slot utilization grows. For example, assuming a slot utilization near to one, say 0.8, we observe that the stations with the highest  $N_A$  value obtains a Probability of Transmission close to one while stations at the first transmission attempt transmit with a probability equal to 0.2.

It is worth noting a property of the DCC mechanism: the slot utilization of the channel never reaches the value One. Assuming  $S_U$  near or equal to One, the DCC mechanism would perform asymptotically (see Figure 4) by reducing the Probabilities of Transmission for every station. This effect was due to the  $P_T$  definition, and in particular to the explicit presence of the upper bound One for the

slot utilization estimate. We will exploit this characteristic of the  $P_T$  definition to construct the AOB mechanism.

### 4. IEEE 802.11 DCF Capacity Analysis results

Since a WLAN relies on a common transmission medium, the MAC protocol coordinates the network stations in accessing the channel by means of control information that is carried explicitly by control messages travelling along the medium (e.g. ACK messages), or can be provided implicitly by the medium itself by the channel being either active or idle (i.e. carrier sensing). Control messages, or message retransmission due to collision, remove channel bandwidth from that available for successful message transmission. Therefore, the fraction of channel bandwidth used by successfully transmitted messages gives a good indication of the overhead required by the MAC protocol to perform its coordination task among stations. This fraction is known as channel utilization, and the maximum value it can attain is known as the *capacity* of the MAC protocol [6].

In this section we briefly present the main results of the IEEE 802.11 capacity analysis developed in [3]. The IEEE 802.11 capacity analysis is performed by assuming an IEEE 802.11 system with M stations working in asymptotic conditions, i.e., each station has always a frame to transmit. The stations transmit frames whose sizes are i.i.d. sampled from a geometric distribution with parameter q. Specifically, the size of a frame is an integer multiple of the slot Size  $(t_s)$ , and hence the Mean Frame Size (MFS) is  $MFS = t_s/(1-q)$ .

To simplify the analysis, in [3], it is assumed a geometrically distributed backoff instead of the uniform sampled backoff of the IEEE 802.11. At the beginning of an empty slot a station starts the transmission of a frame with probability p, and defers the transmission with probability 1-p. Hence, the IEEE 802.11 protocol with the new backoff algorithm is similar to a p-persistent protocol [12].

From the geometric backoff assumption all the processes which define the occupancy pattern of the channel (i.e. empty slots, collisions, successful transmissions) are regenerative with respect to the sequence of time instants corresponding to the completion of a successful transmission. The protocol capacity is thus [14]:

$$\rho_{\max} = \frac{t_{ft}}{t_v} \tag{1}$$

where  $t_{ft}$  is the average Frame Transmission time, and  $t_v$  is the average temporal distance between two consecutive successful transmissions, also referred to as the *average virtual transmission time*. Specifically, the average virtual transmission time includes (see Figure 5):

i) the average time required for a successful transmission,  $t_{st}$ , i.e. the average time

interval including the successful transmission and the overheads induced by the MAC protocol definition. By denoting with  $\tau$  the maximum propagation delay, and with *SIFS*, *ACK*, *DIFS* the corresponding times connected to the protocol implementation (see Table 1), it results [3]:

$$t_{st} \le t_{ft} + 2 \cdot \tau + SIFS + ACK + DIFS$$

- *ii*) the idle periods. An idle period is made up of a number of consecutive slots in which the transmission medium remains idle due to the backoff algorithm;
- iii) the collisions which occur between two consecutive successful transmissions.



Figure 5: Structure of a virtual transmission time

Taking into consideration *i*)-*iii*), it follows that [3]

$$t_{v} = E\left[\sum_{i=1}^{N_{c}} \left(Idle_{p_{i}} + Coll_{i} + \tau + DIFS\right)\right] + E\left[Idle_{p_{N_{c}}+1}\right] + E\left[t_{st}\right] \quad (2)$$

where  $Idle_p_i$  and  $Coll_i$  are the lengths of the *i*-th idle period and collision in a virtual time, respectively; and  $N_c$  is the number of collisions in a virtual time.

The length of a collision is equal to the maximum length of colliding frames (depending on the frame size distribution), and depends on the Backoff Algorithm. The latter determines the number of colliding stations.

As we stated before, we assume that a station for each transmission attempt uses a backoff interval sampled from a geometric distribution with parameter *p*. This means that the average Contention Window size is given by:  $E[CW\_Size] = 1 + 2/p$ .

The assumption that the backoff interval is sampled from a geometric distribution with parameter p implies that the future behavior of a station does not depend on the past. Hence, in a virtual transmission time, i) the idle period times  $\{Idle_p_i\}$  are i.i.d. sampled from a geometric distribution with an average  $E[Idle_p]$ ; and ii) the collision lengths  $\{Coll_i\}$  are i.i.d with average E[Coll]. Thus Equation (2) can be rewritten as

$$t_{v} = E[N_{c}] \{ E[Coll] + \tau + DIFS \} + E[Idle_{p}] \cdot (E[N_{c}] + 1) + E[S] \quad . \tag{3}$$

Closed expressions for  $E[Idle_p]$  and E[Coll] are derived in [3] together with  $E[N_c]$ :

$$E[N_{c}] = \frac{1 - (1 - p)^{M}}{Mp(1 - p)^{M-1}} - 1$$

$$E[Coll] = \frac{t_{slot}}{1 - [(1 - p)^{M} + Mp(1 - p)^{M-1}]} \cdot \left[\sum_{h=1}^{\infty} \left\{ h \cdot [(1 - pq^{h})^{M} - (1 - pq^{h-1})^{M}] \right\} + \frac{Mp(1 - p)^{M-1}}{1 - q} \right]$$

$$E[Idle_{-}p] = \frac{(1 - p)^{M}}{1 - (1 - p)^{M}} \quad .$$

Hence,  $t_v$  is a function of the system's parameters (see Table 1), the number of active stations M, the parameter p which defines the geometric-distribution used in the backoff algorithm, and the parameter q that characterizes the frame-size geometric distribution. Fixed the value for M and q, and the system-parameter values,  $t_v$  is a function of the p value only,  $t_v(p)$ ; therefore by exploiting (3) we can analytically investigate the value of the p parameter that minimizes  $t_v$ , named the optimal p value  $(p_{opt})$ . Hence  $p_{opt}$  is the p value that maximizes the protocol capacity, see Equation (1).

In [3], it is shown that  $p_{opt}$  is closely approximated by the *p* value that guarantees a balance, in a virtual transmission time, between collisions and idle periods, i.e.  $E[Coll] \cdot E[N_c] = (E[N_c]+1) \cdot E[Idle_p] \cdot t_{slot}$ . (4)

#### 4.1 Theoretical limits vs. IEEE 802.11 and DCC protocol capacities

In this section we compare the protocol capacity of the IEEE 802.11 protocol with its theoretical capacity limits derived in the previous section. Specifically, for the capacity of the IEEE 802.11 protocol we consider both the standard protocol (see Section 2) and the protocol enhanced with the DCC mechanism (see Section 3.1).



Figure 6: Channel utilization level of standard 802.11 (with or without DCC) vs. the optimal level.

The results of this comparison are summarized in Figure 6. Results refer to a system with *M* active stations ( $2 \le M \le 200$ ), and with the physical-parameter values defined in Table 1. The capacity results have been derived by assuming that the length of messages is sampled from a geometric distribution with parameter *q*. Furthermore, according to the studies of TCP traffic [18] we consider either systems with "*long messages*" (average length of 100 slot times), or systems with "*short messages*" (average length of 2.5 slots). Results related to the IEEE 802.11 protocol with or without the DCC mechanism have been obtained via simulation.<sup>3</sup>

Figure 6 shows that the DCC mechanism effectively enhances the channel utilization of the standard protocol. However, its capacity is still far from the optimal capacity. It should be noted that the growth of the contention level (number of active stations) implies a reduction of the channel utilization in the IEEE 802.11 protocol (with or without the DCC mechanism). This reduction is more marked when long messages are transmitted due to the high collision cost.

To summarize, results presented in this section show that the contention reduction introduced by the DCC mechanism is effective. However, DCC operates in a heuristic way. Its aim is to utilize, when the network congestion increases, larger windows respect to the standard sizes, but DCC does not have any "idea" of the optimal window size given a contention level. Other approaches have been proposed in the literature to dynamically tune the backoff window size [1, 3]. The main limitation of these approaches is the need to estimate from the network the

<sup>&</sup>lt;sup>3</sup> The performance indices have been estimated with the independent-replication technique (confidence level 90%, and the width of the confidence interval is about 3-5%).

information on the contention level. On the other hand the main advantage of the DCC approach is its simplicity, low cost, adaptiveness to network congestion levels and transparency with respect to the standard protocol.

In the next section we define a mechanism named AOB (Asymptotically Optimal Backoff) that dynamically tunes the backoff window size to the optimal values still maintaining the main advantages of the DCC mechanism. AOB exploits some observations that can be derived from results presented in Figure 6:

*i*) the increase in the number of active stations have an almost negligible impact on the theoretical capacity bounds;

*ii*) the payload size highly affects the optimal utilization level. Specifically, decreasing the payload size implies a reduction of the optimal utilization level. This can be expected as reducing the payload size produces a percentage increase of the transmission overheads.

# 5. The AOB mechanism

In this section we exploit the results obtained from the analysis of the theoretical capacity limits of the IEEE 802.11 protocol to develop the AOB mechanism. The aim of this mechanism is to dynamically tune the backoff window size to achieve the theoretical capacity limit of the IEEE 802.11 protocol.

The proposed AOB mechanism is simpler, more robust and with lower costs and overheads introduced than the contention mechanism proposed in [3]. Specifically, the AOB mechanism does not require any estimate of the number M of active stations. Moreover, the AOB mechanism can be used in an IEEE 802.11 station without any modification to the standard protocol.

# 5.1 Theoretical capacity limits: an invariant figure

In Section 4 we have pointed out that the increase in the number of active stations have an almost negligible impact on the theoretical capacity bounds, while the payload size highly affects the optimal utilization level. Results presented in Table 2, derived from Formulas (3) and (4) explain these effects. In the table we report for various network (number *M* of active stations) and traffic (message length) configurations the analytical values of the optimal  $p_{opt}$  parameter, i.e., the *p* value minimizes the  $t_v$  expression (3) given the *M* and *q* values. In the table we also report for each configuration the value  $M \cdot p_{opt}$ . It is worth noting that while  $p_{opt}$  is highly affected by the *M* value, given a *q* value, the product  $M \cdot p_{opt}$  is almost constant. This is the reason for naming it as an *invariant figure*. As we explain below,  $M \cdot p_{opt}$  is a measure of the network contention level when the network utilises the optimal window size corresponding to the ongoing network and traffic configuration.

**Table 2** : Analytical definition of optimality in function of M and q

q	MFS	M = 2		M = 4		<i>M</i> = 10		<i>M</i> = 50		<i>M</i> = 100	
values	(Slots)	$p_{opt}$	$M \cdot p_{opt}$	$p_{opt}$	$M \cdot p_{opt}$	$p_{opt}$	$M \cdot p_{opt}$	$p_{opt}$	$M \cdot p_{opt}$	$p_{opt}$	$M \cdot p_{opt}$
0.5	2	.26160	.52321	.11679	.46715	.04430	.44304	.00864	.43206	.00431	.43076
0.9	10	.18260	.36521	.07880	.31520	.02945	.29448	.00570	.28518	.00284	.28409
0.96	25	.13293	.26586	.05638	.22552	.02091	.20914	.00404	.20186	.00201	.20101
0.98	50	.10053	.20106	.04221	.16883	.01559	.15591	.00300	.15018	.00149	.14952
0.98	82	.08119	.16239	.03389	.13557	.01249	.12490	.00240	.12018	.00120	.11963
0.99	100	.07434	.14868	.03097	.12388	.01140	.11403	.00219	.10968	.00109	.10918

In the following we investigate the meaning of  $M \cdot p_{opt}$ . As defined in Section 4, we consider *M* active stations scheduling their transmission attempts in a slot selected according to a geometric distribution with parameter *p*. Furthermore, for each configuration it exists an optimal value of parameter *p*,  $p_{opt}$ , that guarantees the balancing on the channel between idle periods and collisions. Let us now assume that each station uses the optimal value  $p_{opt}$ .

In Section 3 we introduce the *slot utilization*  $(S_U)$  parameter to estimate the network contention level. Let us now investigate the relationship, in the tagged contention window, between  $S_U$  and  $M \cdot p_{opt}$ .

We denote with  $N_{tr}$  the number of stations that make a transmission attempt in a slot. Hence  $P\{N_{tr} = i\}$  is the probability that exactly *i* stations transmit in a slot, and  $P\{N_{tr} = 0\}$  is the probability that a slot remains empty. Let us now observe that  $M \cdot p_{opt}$  is the average number of stations which transmits in a slot:

$$M \cdot p_{opt} = \sum_{i=1}^{M} i \cdot P\{N_{tr} = i\} \ge 1 - P\{N_{tr} = 0\} = S_U$$
(5)

hence  $M \cdot p_{opt}$  is an upper bound on the probability to observe a busy slot, i.e.,  $M \cdot p_{opt} \ge S_U$ . Furthermore, results presented in [3] indicate that, if the stations utilize the optimal p value, the collision probability is low (e.g. in average one collision occurs out of several virtual times). This means that

$$P\{N_{tr} > 1 | N_{tr} > 0\} << P\{N_{tr} = 1 | N_{tr} > 0\}$$

and hence from (5) it results that  $M \cdot p_{opt}$  is a tight upper bound of  $S_U$  in a system operating with the optimal channel utilization level.

#### 5.2 Considerations about the optimal slot utilization level

In the previous section we show that, given the q value, in the "ideal case" (i.e. the stations tune the window size according to the optimal p value) the slot utilization level is bounded by  $M \cdot p_{opt}$ . Now, we compare the slot utilization level of the ideal case with that of the standard IEEE 802.11 with or without the DCC mechanism. We define the optimal slot utilization values exploiting the previous analyses and considerations. Specifically, we characterize the  $S_U$  in the ideal case by its upper bound  $M \cdot p_{opt}$  (see Table 2).



Figure 7: The steady-state slot utilization: DCC, standard 802.11 and optimal values

In Figure 7, we compare the optimal  $S_U$  values with the steady-state slot utilization level estimated (via simulation) in an IEEE 802.11 network with or without the DCC mechanism. It is worth noting that by adopting the IEEE 802.11 protocol the slot utilization level does not depend on the parameter q value, while it is connected only to the number of active stations, M.

Results reported in Figure 7 show that in the standard protocol the  $S_U$  values are generally greater than the optimal values. This overestimation of the optimal  $S_U$  values is marked when the mean frame size is large (frame size greater than 50 slots in the figure). This can be expected because the standard protocol produces a slot utilization level which does not depend on the frame size. On the other hand, in the optimal case, the increase of the frame size which means an increase in the collision-cost that is balanced by a decrease of the collision probability. Obviously, the decrease of the collision probability is achieved (in the optimal case) by decreasing the  $S_U$  value. Even if the DCC mechanism correctly reduces, with respect to standard 802.11, the slot utilization (i.e. the contention level) under high-load conditions, the results presented in Figure 7 indicate that DCC does not produce the optimal channel utilization level. Furthermore, these results indicate that an algorithm which wish to drive the system to the optimal channel utilization must take into consideration the value of the q parameter.

### 5.3 The AOB mechanism

Let us coming back to the results presented in Table 2. Specifically, by fixing a given value for the frame size parameter q, it can be observed that the  $M \cdot p_{out}$  product

results quasi-constant for M greater than 2, and more precisely, it shows only a little decrease when M grows. This means that, fixed the system's parameters of Table 1, it is possible to define a single quasi-optimal value for the  $M \cdot p_{opt}$  as a function of the single parameter q. To sum up, for each IEEE 802.11 physical layer parameters setting (e.g. see Table 1), it is possible to define a function of q, named Asymptotical Contention Limit, ACL(q), such that  $ACL(q) \approx M \cdot p_{opt}$ ,  $q \in [1/2, 1]$ . This function would represent the optimal slot-utilization level the system should obtain to guarantee its optimal behavior from the channel utilization viewpoint. The ACL(q) function can be computed off-line by exploiting the analytical model presented in Section 4. It is worth noting that ACL(q) identifies the optimal contention level without requiring the knowledge of the number of active stations in the system. This is important, because it is the basis for implementing an optimal window-tuning mechanism which does not require to estimate the number of active stations in the system. Specifically, in the following we show how to limit the slot utilization by exploiting the Asymptotic Contention Limit. The basic idea is i) to utilize, as in the DCC mechanism, the slot utilization level to control the stations' transmission, and *ii*) to disable the stations' transmissions when the slot utilization level is greater or equal to ACL(q).

As far as point *i*) is concerned it is worth remembering that in DCC the control on the station's transmission via the slot utilization is obtained by introducing a Probability of Transmission ( $P_T$ ):

$$P_T(S_U, N_A) = (1 - S_U^{N_A})$$

As discussed in Section 3.1 the P\_T defined above guarantees that the S\_U asymptotically tends to 1. The asymptotic effect is due to the P\_T definition which implies that P\_T tends to zero as the slot utilization approaches one. Intuitively, if the slot-utilization boundary value (i.e. one for DCC) would be replaced by the ACL(q) value, we reduce all the probabilities of transmission to zero in correspondence of slot utilization values greater or equal to the ACL(q). To this end in the AOB mechanism we introduce a new definition for the Probability of Transmission :

$$P_T(ACL, S_U, N_A) = 1 - \min\left(1, \frac{S_U}{ACL}\right)^{N_A}$$
 (6)

Fixed a given ACL(q) value, the P\_T values obtained fluctuates among 0 and 1. We named *Asymptotically Optimal Backoff* (AOB) a mechanism which, by using the P\_T defined by Equation (6), guarantees a  $S_U$  value below the given ACL(q) value. The optimal ACL value for the slot utilization could be only asymptotically approximated, for this reason the mechanism is named Asymptotically Optimal Backoff.



Figure 8: ACL(q) exact vs. approximation

# 6. Implementing the AOB Mechanism

In the previous section we introduce the AOB mechanism. This mechanism requires the knowledge of the q value to identify the Asymptotic Contention Limit ACL(q) for the slot utilization. The ACL(q) value together with the current  $S_U$  estimate determines the transmission probability. In the following subsections we show how the AOB mechanism can be implemented.

# 6.1 ACL(q)

As we said before ACL(q) provides, given a q value, an upper bound of  $M \cdot p_{opt}$ . Furthermore,  $M \cdot p_{opt}$  tends asymptotically with the increase of M, to ACL(q). Hereafter we use as the asymptotic value of  $M \cdot p_{opt}$  the value corresponding to M=100. Anyway, from Table 2, it can be observed that this  $M \cdot p_{opt}$  value is a good approximation even for a limited number of stations. In principle, for each q value we can compute the  $M \cdot p_{opt}$  as shown in Section 4 and then we can store in the AOB mechanism the corresponding asymptotic value. As the number of interesting q values could be large, this approach would require a large storage space. This complexity can be avoided by defining an analytical ACL(q) expression which interpolates the asymptotic  $M \cdot p_{opt}$  values. This function, by supplying approximated  $M \cdot p_{opt}$  values for each possible value of the parameter q, will constitute the instrument to make the AOB mechanism pursuing the optimal channel utilization level for each system's configuration. Figure 8 shows the values for the product  $M \cdot p_{opt}$  obtained for the system considered in Table 1, with M = 100 stations. In the same figure are also reported the values of an approximated function ACL(q) used in the simulations we performed:  $ACL(q) = 0.6 \cdot (1 - q^{1.3})^{0.333}$ . It must be pointed out that, even though the above ACL(q) expression provides a rough approximation of the  $M \cdot p_{opt}$  values, it provides a low-complexity and low-cost procedure to compute at run time the bounds on the slot utilization. This is an interesting property both for the implementation of the AOB mechanism, and for simulative studies of its performance.

### 6.2 q Estimation

Here we present an estimate for the parameter q, which is simple and it has low cost, i.e., no overheads or additional hardware introduced with respect to the actual 802.11 standard requirements. In fact, this could be performed by each station simply on the basis of the Carrier Sensing mechanism adopted for the accesses to the channel. The q parameter estimate is based upon a simple and progressive approximation criterion that exploits the consecutive observations of the Frame Size Estimate (*FSE*). Specifically, the *FSE* estimate is simply constituted by the total transmission time observed during a backoff period, divided by the number of transmissions observed<sup>4</sup>. Each time a station schedules a transmission attempt for a tagged frame, say the *i*-th transmission attempt, it initializes a DCF clock counter. While the backoff period elapses, the station performs the Carrier Sensing activity (physical or virtual according to the Standard [8]) and counts the number of observed transmissions (i.e., periods during which the channel is consecutively in the busy state separated by at least a DIFS idle period). When the Backoff Counter reaches the value Zero, then the station obtains the actual *FSE*<sup>(i)</sup> estimate as:

$$FSE^{(i)} = \frac{Clock^{(i)} - \left[Num\_tr^{(i)} \cdot (Header + SIFS + ACK + DIFS)\right] - Init\_Backoff^{(i)} \cdot Slot\_Time}{\max(1, Num\_tr^{(i)})}$$

where :

- $Clock^{(i)}$ : it is the actual value of the DCF clock counter started at the *i*-th schedulation time. The clock runs only when the protocol operates in the DCF mode.
- $Num_{tr}^{(i)}$ : it is the counter for the total number of transmission observed in the *i*-th Backoff Interval (all assumed to be successful).
- *Init\_Backoff*<sup>(*i*)</sup>: it is the value for the Backoff Counter at the beginning of the *i*-th transmission attempt of a frame.

<sup>&</sup>lt;sup>4</sup>In the real case, the clock should be frozen while transmitting the frames of a multiple frame sequence (see [8]), except the first one. This would happen because the mean frame size of frames subject to contention would be affected only by the first one. Here we assume only single frame's transmissions.

The given  $FSE^{(i)}$  value obtained after the *i*-th Backoff Interval is then used to obtain the next approximation of the *q* estimate, by means of a weighted expression. The relative weights are given by the number of transmission observed during last backoff period with respect to the total number of transmissions observed from the first time the tagged frame has been scheduled for transmission. To do this, the given  $FSE^{(i)}$  value has to be converted from the absolute time scale into the *q* parameter relative scale, as:

$$q_{last}^{(i)} = 1 - \frac{Slot\_Time}{FSE^{(i)}}$$

The proposed expression for the *i*-th successive approximation, given the previous (i-1)th estimate, is then:

$$q_{est}^{(i)} = q_{last}^{(i)} \cdot \alpha^{(i)} + q_{est}^{(i-1)} \cdot (1 - \alpha^{(i)}) \quad ,$$

where

$$\alpha^{(i)} = \frac{Num_t r^{(i)}}{\max\left(1, \sum_{k=1}^{i} Num_t r^{(k)}\right)}$$

The proposed expression for the weight has been adopted to make the influence of the *i*-th estimate dependent from the significance level achieved by the estimate, i.e. the number of transmissions observed. For example, if the (i-1)th estimate is based on few transmissions, and the i-th estimate is based on the observation of a significant number of transmissions, then our  $q_{est}^{(i)}$  estimate is weighted in a correct way. Moreover, the proposed estimate is stable because the relative weight of further observations becomes lower and lower.

Let us make some comments on the q parameter estimation algorithm. It has to be noted that, in the *FSE* expression, it has been assumed that each observed transmission resulted as successful. This fact is useful to avoid a further effort caused by the need to observe the result (success or collision) of a transmission. However, we need to take into consideration that, due to collisions' occurrence, our assumption results in a distortion of the q parameter estimate. Given the collisions' influence, the average time the channel is busy results greater than the mean frame size of transmitted frames. This implies that each time one (or more) collision occurs, the q parameter estimate results overestimated. This would force the AOB mechanism to use a greater value of q (with respect to the real value), and hence an ACL(q) which corresponds to a lower limit for the slot utilization on the channel.<sup>5</sup> This is however the reaction we could desire from the system. Indeed when the collision rate increases, thus producing a biasing on the q estimate (i.e. the average

<sup>&</sup>lt;sup>5</sup>In fact, the ACL(q) is a monotone decreasing function of q.

message length is overestimated), the AOB mechanism reduces the network contention level by decreasing the slot utilization level on the channel.

### 6.3 *S*\_*U* Estimate

The *S*\_*U* estimate procedure operates independently in each station with packets ready for transmission (*busy station*). The procedure starts, with a conservative assumption, when the status of the station changes from idle to busy. Then the *S*\_*U* estimate is refined through successive approximations. Specifically, the initial *S*\_*U* value, say  $S_U^{(0)}$ , is set to ACL(q), where the *q* parameter value is obtained from the size of the first frame to transmit:  $S_U^{(0)} = ACL(q)$ .

The *S*\_*U* estimate is refined after each backoff interval. Specifically, a *S*\_*U* measure obtained in the i-th backoff interval,  $S_{-}U_{measure}^{(i)}$ , contributes to the slot utilization estimate with a weight proportional to the number of slots observed in that backoff interval:

$$S_{U^{(i)}} = S_{U^{(i)}_{measure}} \cdot \beta^{(i)} + S_{U^{(i-1)}} \cdot \left(1 - \beta^{(i)}\right)$$

where

$$\beta^{(i)} = \frac{Init\_Backoff^{(i)}}{CW\_Size\_MAX}$$

This  $\beta^{(i)}$  definition roughly limits to the last  $CW\_Size\_MAX$  slots the meaningful events for the current estimate.

#### 7. Simulation results

In this section, by means of the discrete event simulation, we investigate the performance of the IEEE 802.11 protocol enhanced with the proposed AOB mechanism. Simulation is performed by exploiting the RESQ simulation tool [16].

The main target of this performance study is to investigate the relationship between the channel utilization level and the network contention.

To perform this study we run a set of simulative experiments in which we change the number M of active stations. Active stations are assumed to operate in asymptotic conditions (i.e., with continuous transmission requirements). We use a maximum number of 200 active stations because the number of stations expected in the future for such a system could raise the order of hundreds [4]. Using up to 200 active stations enable us to emphasize the system's characteristics, adaptiveness and scalability. The physical characteristics and parameter values of the investigated system are reported in Table 1.

It is also interesting to note that other interesting performance indices as the Throughput and the Mean Access Delay are strongly correlated with the channel utilization level.



Figure 9.a: IEEE 802.11 channel utilization vs. optimal value

### 7.1 Channel utilization level

The Figure 9.a shows the channel utilization level of the standard 802.11 DCF with respect to the optimal values calculated by means of the analytical model defined in Section 4. For a given system configuration, the optimal channel utilization value has been obtained by computing  $p_{opt}$  as explained in Section 4. It is immediate to verify that the performance of the IEEE 802.11 standard protocol are negatively (low channel utilization) affected by high-contention situations. In fact, with the standard protocol the channel utilization level decreases when the contention grows; this implies that collisions and retransmissions reduce the amount of user data which is possible to deliver. Note that this problem occurs for each possible value of the mean payload size considered.

The effectiveness of the proposed AOB mechanism is shown in Figure 9.b. This figure shows the channel utilization level achieved by adopting the AOB system and compares this index with the analytically-defined optimal utilization levels. The results show that the AOB mechanism leads an IEEE 802.11 network near to its optimal behavior at least from the channel utilization viewpoint. Only a little overhead is introduced when only few stations are active, as we can see in the direct comparison presented in figure 9.c. Moreover, with the AOB mechanism, the channel utilization remains close to its optimal value even in high-contention situations. In such cases, AOB almost doubles the channel utilization with respect to the standard protocol.



Figure 9.b: Channel utilization of the IEEE 802.11 protocol with the AOB mechanism vs. optimal value

### 7.2 The 99-th percentile of MAC access delay.

The channel utilization provides information about the efficiency of a MAC protocol in sharing the channel among several stations. However, to measure the network QoS from the users' standpoint other performance indices must be used. The delay a user experience to transmit a packet is generally used to estimate the QoS a user can rely upon. In this section we utilize the MAC delay, i.e. the time interval between the first time a packet is scheduled for transmission and the instant at which its successful transmission is completed.

In Figure 10 we report the 99-th percentile of the MAC delay vs. contention level (i.e. number of active stations) for various average sizes of the transmitted frames. It results that the AOB mechanism leads to a great reduction of the worst case MAC delay with respect to the standard access scheme alone. This gives also a good indication of the reduced risk of starvation for transmissions. The AOB mechanism, by exploiting the priority effect induced by the  $N_A$  parameter used in the probability of transmission, increases the stations'  $P_T$  with the increase of MAC delay. This behavior enhances the fairness and the queue-emptying behavior of the system.



Figure 9.c: IEEE 802.11 channel utilization with or without the AOB mechanism



By a careful observations of Figures 9c and 10 it is clear that the  $N_A$  priority mechanism is really effective in reducing the tail of the MAC Delay. For example, for average payload equal to 100 slots, the ratio between the 99-th percentile of the MAC Delay with or without the AOB mechanism is about 6 while the ratio between the average MAC Delay is about 2. Note that the average MAC Delay ratio is exactly the inverse of the channel utilization ratio (see Figure 9c).

### 8. Conclusions and future research

In this paper we have proposed and evaluated the AOB mechanism which can be applied on top of an IEEE 802.11 network to dynamically control the network contention level. This mechanism leads to the optimal channel utilization level in a fully distributed way. The network contention level is measured independently by each stations, by an index simple to estimate (the slot utilization), and it has a feedback effect on the station behavior. This effect is obtained through the definition in each station of a probability of transmission which is a function of the ratio between the measured slot utilization level and an optimal utilization level (i.e., the utilization level which guarantees the maximum channel capacity). The optimal utilization level has been analytically derived, and we have shown in the paper that, for a given system configuration, it is significantly affected only by the average size of transmitted frames. Hence to implement the AOB mechanism we need to estimate both the slot utilization and the average frame size. Both estimates can be done with low cost and with no overheads or further hardware introduced with respect to the standard protocol.

The AOB mechanism can be used on top of the standard 802.11 DCF, allowing a quasi-optimal sizing of the contention window without paying (as it occurs with the standard Binary Exponential Backoff mechanism alone) any collision cost. A great improvement in the channel utilization is thus obtained, which results almost uninfluenced by the contention level in the system. AOB always maintains the system near to its optimal behavior (from the channel utilization standpoint). From the user standpoint, the age information introduced in AOB via the  $N_A$  (Number of transmission attempts) parameter, leads to a significant reduction of the 99-th percentile of the Access Delay.

Traffic with different priorities' levels can be also easily introduced in an IEEE 802.11 network with the AOB mechanism. This aspect is currently under investigation. Two directions are investigated *i*) the extension of the probability of transmission with the Prior\_Lev parameter, and *ii*) the use of different ACL(q) functions depending on the size of the frame to transmit.

Future research involves the study of the use of the AOB mechanism to introduce power-saving policies (exploiting the reduction of collision cost obtainable with AOB), and the investigation of the AOB advantages in the transmission of prioritized RTS/CTS messages.

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